# Non-linear Partial differential eqn of 1st order but of any degree

General methoed for these non-linear parial differential equation is charpits methed.

Befor giving charpits methed we can solve easily by typical (special types) metheds.

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**Shorter Methed:→**

**Type-I:→** Eequation of the form

$f(P, q) = 0$ . & not existing $x, y, z$

Here the complete integral is obtained by $z = ax + by + c$, Where a & b are connected

by $f(a, b) = 0$ & $p = \frac{\partial z}{\partial x} = a$,

$q = \frac{\partial z}{\partial y} = b$

(It will clear from charpits methed)

Example

(Q1) $p^2 + q^2 = 1$.

Soln:→ It is $f(P, q) = 0$ & having no, $x, y, z$, solution is given by (as trial solⁿ)

$z = ax + by + c$, where $p = a$, $q = b$

i.e $a^2 + b^2 = 1$

$\therefore b = \sqrt{1-a^2}$

Here complete integral is

$$Z = ax + \sqrt{1-a^2}\, y + c$$

For general integral put $c = \phi(a)$

$$\therefore Z = ax + \sqrt{1-a^2}\, y + \phi(a)$$

differenceating w.r.to a, we get

$$0 = x + \frac{-a}{\sqrt{1-a^2}}\, y + \phi'(a)$$

eliminating (1) we get general solⁿ.

Q2) $p^2 - q^2 = 1$

solⁿ:→ The solution is

$z = ax + by + c$, where

$a^2 - b^2 = 1$

$\therefore b = \sqrt{a^2 - 1}$

Here complete solⁿ is $z = ax + \sqrt{a^2-1}\, y + c$

Put $c = \phi(a)$ & differenceating w.r.to a eliminating 'a' to get general solⁿ.

(3) $p^2 + q^2 = n^2$

sol$^n$: → complete sol$^n$ is $z = ax + by + c$,

Where $a^2 + b^2 = n^2$

$\therefore b = \sqrt{n^2 - a^2}$

$$\therefore z = ax + \sqrt{n^2 - a^2}\, y + c$$

Let $c = \phi(a)$

$\therefore z = ax + \sqrt{n^2 - a^2}\, y + \phi(a)$ — (2)

Differenceating (1) w.r.to 'a' we get

$$0 = a + \frac{-a}{\sqrt{n^2 - a^2}}\, y + \phi'(a) \quad -(3)$$

eliminating 'a' from (1) & (2) we get the general sol$^n$.

(Q4) $p^2 + q^2 = npq$

sol$^n$: → Which is of the form $f(p, q) = 0$

$\therefore$ compete integral is

$$z = ax + by + c$$

Where $a^2 + b^2 + nab$

$$\Rightarrow b^2 - (na)\, b + a^2 = 0$$

$$\therefore b = \frac{na \pm a\sqrt{n^2 a^2 - 4a^2}}{2}$$

$$= \frac{na \pm a\sqrt{n^2-4}}{2}$$

$$= \frac{a}{2}\left(n \pm \sqrt{n^2-4}\right)$$

$\therefore$ complete integral is

$$Z = ax + \left\{ \frac{a}{2}\left(n \pm \sqrt{n^2-4}\right) \right\} y + c$$

Q) $x^2p^2 + y^2q^2 = z^2$

sol$^n$:→ $\therefore x^2\left(\frac{\partial z}{\partial x}\right)^2 + y^2\left(\frac{\partial z}{\partial y}\right)^2 = z^2$

Put

$$\therefore \left(\frac{x}{z}\frac{\partial z}{\partial x}\right)^2 + \left(\frac{y}{z}\frac{\partial z}{\partial y}\right)^2 = 1 \quad — (1)$$

Put $\frac{1}{z}dz = dZ$ i,e $z = e^Z$

$\frac{1}{x}dx = dX$ i,e $x = e^X$

$\frac{1}{y}dy = dY$ i,e $y = e^Y$

$\therefore$ eq$^n$ (1) becomes

$$\left(\frac{dZ}{dX}\right)^2 + \left(\frac{dZ}{dY}\right)^2 = 1$$

Which is the form of $f(P,q)=0$

$\therefore$ Complete integral is

$$Z = ax + by + c_1$$

Where $a^2+b^2=1$

$a, \; b=\sqrt{1-a^2}$

$$\therefore Z = ax + \sqrt{1-a^2}\, y + c_1$$

$$\therefore \log z = a \log x + \sqrt{1-a^2} \log y + c_1$$

(Q) $p^2+p=q^2$

Sol$^n$: → It is of the form of $f(P,q)=0$

$$\therefore Z = ax + by + c$$

Where $a^2+a=b^2$

$$\Rightarrow b = (a^2+a)^{1/2}$$

$$\therefore Z = ax + (a^2+a)^{1/2} + c$$

(8) $p + q = pq$

Sol$^n$:→ It is of the form of
$f(p, q) = 0$

∴ compete integral is

$$z = ax + by + c$$

Where $a + b = ab$

$$\Rightarrow a = ab - b = b(a-1)$$

$$\therefore b = \frac{a}{a-1}$$

∴ compete integral is

$$z = ax + \frac{a}{a-1} y + c$$